LEAKAGE FIELDS FROM PLANAR SEMI-INFINITE TRANSMISSION LINES

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ABSTRACT

The leakage fields radiated by planar semi-infinite transmission line currents are studied for two canonical structures, the air-gap stripline and a strip in free space. The exact leakage fields are compared with the following two asymptotic approximations: a stationary-phase (geometrical optics or GO) evaluation and a uniform asymptotic expansion.

I. INTRODUCTION

The excitation of leaky modes on planar printed transmission lines is undesirable because it results in radiated power loss and can cause unwanted interference (crosstalk) between adjacent lines. Although the leakage field has been characterized in the past for infinite-length lines [1]-[3], no such study has been undertaken for the more practical case of a leaky mode on a semi-infinite line (the type of wave excited by a feed). It is therefore important to characterize the leakage fields excited by semi-infinite lines. To aid in this characterization, we derive accurate, closed-form expressions for the leakage fields from two canonical structures, the air-gap stripline structure with a semi-infinite strip, and a semi-infinite strip in free space. Two types of asymptotic expressions for the fields are also derived.

A. Background Information

Figure 1 shows an endview of the air-gap stripline structure employed in the analysis. This structure is representative of a general class of printed circuit lines where a leaky mode radiates into the fundamental background mode of the structure. The propagation constant of the leaky mode is denoted as $k_{z0} = \beta_z - j\,\alpha_z$.

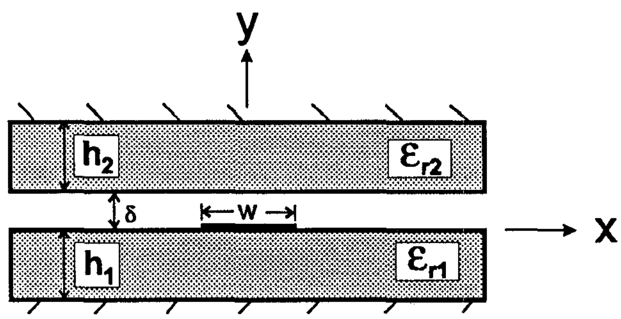

Fig. 1. Endview of the air-gap stripline structure. In the present analysis, $h_1 = h_2 = w = 0.133\lambda_0$, *where* $\lambda_0 = 7.5$ *mm at f=40 GHz, and* $\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_r = 2.2$. *The air-gap thickness is* $\delta = 0.0133\ \lambda_0$. *The normalized propagation constant of the* TM_0 *mode is* $k_{TM_0} / k_0 = 1.4436$.

The classical transverse field behavior of the leaky mode on this structure is based on the simple ray picture shown in Fig. 2. This simple model predicts that the leakage field has an amplitude that varies as $\exp(|\alpha_x|\ x)$ in the transverse direction, out to the point $x_0 = z \tan\theta$ (as seen in Fig. 2), where the leakage angle θ is

$$\theta \cong \theta_0 \equiv \cos^{-1}(\beta_z / k_{TM_0}). \tag{1}$$

The approximation above is accurate when $\alpha_z << \beta_z$ and $\beta_z < k_{TM_0}$. For $x > x_0$ the fields are essentially zero according to this simple model.

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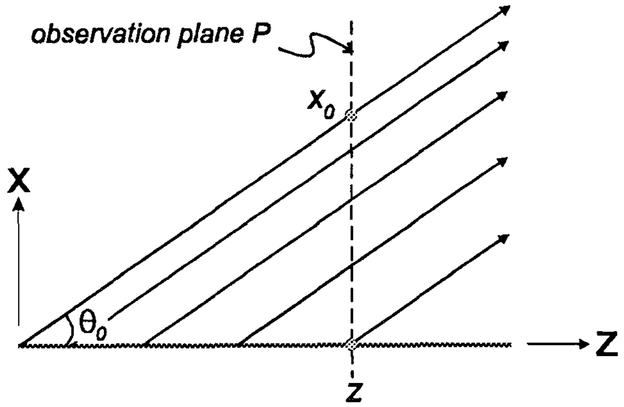

Fig. 2 The ray model of the leakage field, showing the boundary of the leakage region (point x_0) for a given observation distance z from the feed.

B. Anomalous Leakage Behavior

Figure 3 shows the $\hat{y}$-component of the electric field versus a normalized transverse distance $x_n \equiv x / x_0$. The field is obtained using a full-wave spectral-domain formulation for the total field of the semi-infinite strip current,

$$J_s = \hat{z} e^{-jk_{z0} z} \qquad z > 0, \quad |x| < w/2 \tag{2}$$

(the details of this calculation are omitted).

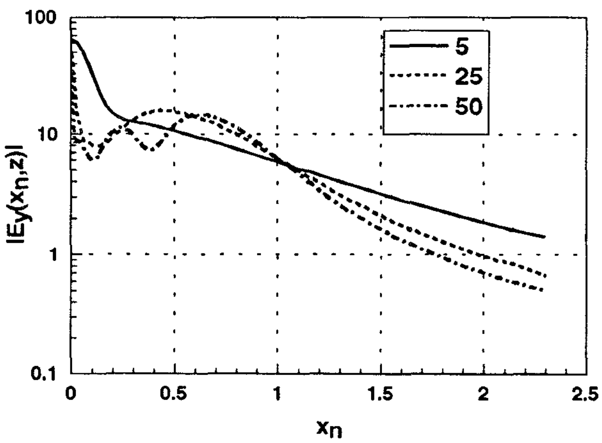

Fig. 3 The magnitude of the y-component of the total electric field, E_y (x_n, z), versus x_n for the semi-infinite air-gap stripline of Fig. 1 at z / λ_0 = 5, 25, and 50 with k_{z0} / k_0 = 1.396 - j0.002. The electric field is normalized by multiplying with the substrate thickness h.

Even with this relatively small value of α_z, the field oscillates in the region $x_n < 1$, and does not peak at the expected location (x_n = 1) for any of the three different values of z / λ_0 shown. Hence, the fields do not exhibit the behavior predicted by the simple ray model. This observation motivated the detailed study presented here.

II. LEAKAGE FIELD OF THE AIR-GAP STRIPLINE

A. Parallel-Plate Mode Leakage Formulation

For ease of computation, the radiated parallel-plate mode (PPM) fields are calculated in lieu of the total fields (this is accurate provided the distance from the strip in the transverse direction is larger than approximately one wavelength). The radiated PPM field is found by using the Green's function for the PPM field radiated by a horizontal electric dipole and integrating over the strip current. The result is

$$E_y^{pp}(x,y,z) = A \int_{-w/2}^{w/2} \int_0^{\infty} H_1^{(2)}(k_{pp}\rho) \cos\varphi \; J_{sz}(x',z')\, dz'\, dx' . \tag{3}$$

B. Stationary-Phase and UAE Asymptotic Evaluations

An asymptotic evaluation of Eq. (3) is performed in the limiting case $z / \lambda_0 \to \infty$ while the product $p = (\alpha_z z)$ remains finite. Applying the conventional stationary-phase (SP) method yields the asymptotic field

$$E_y^{pp} \approx \frac{j2A}{k_{TM_0}} \cot\theta_0 \; e^{-j k_{z0} z'_0} \; e^{-jk_{TM_0}\sqrt{x^2 + (z'_0 - z)^2}} . \tag{4}$$

In analogy with optics, the above result is termed the geometrical optics (GO) approximation of the leaky-wave field. The physical interpretation of this result is that the leakage field at any observation point is mainly due to a single ray, which travels down the line with a wavenumber k_{z0} before shedding off toward the observation point (at the leakage angle θ_0), traveling with a wavenumber k_{TM_0} .

A more accurate asymptotic evaluation of Eq. (3), which accounts for the diffraction-like oscillations in the leakage field, and is continuous across the leakage shadow boundary, may be obtained by employing a uniform asymptotic expansion (UAE) as in diffraction theory (see for example [4]). This analysis gives rise to an asymptotic solution involving the Fresnel integrals, similar to that encountered in diffraction theory. The UAE yields results that are significantly more accurate than the SP method, even at relatively small values of z / λ_0 (the details are omitted here).

III. LEAKAGE FROM THE SEMI-INFINITE LINE IN FREE SPACE

A strip of impressed current having the form $J_{sz}(z') = \exp(-jk_0 z')$ for $z > 0$ is assumed to reside in free space. This artificial current is representative of any structure where leakage occurs into *space*. The strip current is convolved with the appropriate Green's function for a horizontal dipole (HED) to yield the co-polarized (z-directed) and cross-polarized (y-directed) fields. The geometry is the same as in Fig. 1 except that the substrates and ground planes are removed. The asymptotic analysis (UAE) follows a similar line of reasoning as discussed in Sect. IIB (the details are omitted here).

IV. RESULTS

A. Air-Gap Stripline

Figure 4 shows the UAE results for the PPM field of the stripline versus x_n with $z / \lambda_0 = 10^3$ and $p \equiv \alpha_z z / (2\pi) = 0.1, 0.5, 1.0$, and 2.0. In this figure, it is notable that the field in the lit region ($x_n < 1$) is exponentially increasing for the two lower values of p, and is GO-like (smooth with little oscillations) for the lowest value of p. However, the field begins to show an anomalous effect for the larger values of p. The relatively *flat* portions of the $p = 1.0$ and $p = 2.0$ curves in the figure are due to a *near-field* effect in which the large attenuation on the strip causes it to appear effectively as a finite-length antenna localized near the origin. This near-field effect dominates the exponential leakage behavior for transverse distances relatively close to the strip.

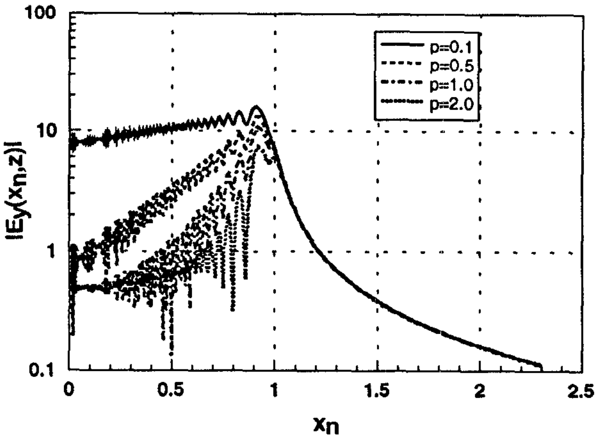

Fig. 4. The UAE results for the PPM field of the semi-infinite air-gap stripline of Fig. 1 versus x_n with $\beta / k_0 = 1.396$, $z / \lambda_0 = 1000$, and $p = \alpha_z z / (2\pi) = 0.1, 0.5, 1.0$, and 2.0. The electric field is evaluated at $y = -h$ and is normalized by multiplying by the substrate thickness h.

B. Line in Free Space: Co-Pol

Figure 5 shows the co-pol field of the semi-infinite line versus x_n with $z / \lambda_0 = 10^3$ and $p = 0.1, 0.5, 1.0$, and 2.0.

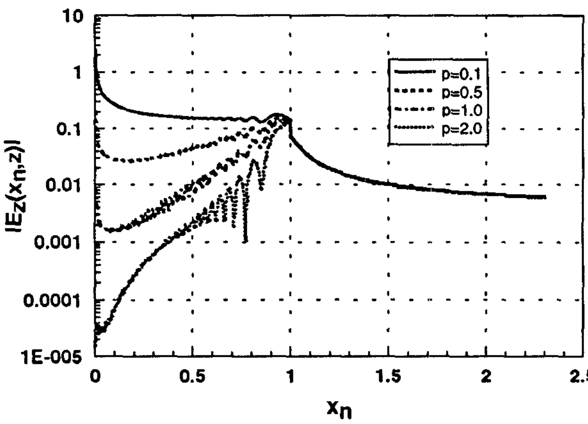

Fig. 5. The UAE co-pol field of the semi-infinite line source in free space versus x_n for $p = \alpha_z z / (2\pi) = 0.1, 0.5, 1.0$, and 2.0, with $z / \lambda_0 = 1000$, $k / k_0 = 1.0$, and $\beta / k_0 = 0.967$. The electric field is evaluated at $y = -h$ and is normalized by multiplying by the distance $h = 0.1 \lambda_0$.

Here, a GO-like behavior is observed for smaller values of p, although the field is not an exponentially increasing one as predicted by the simple ray model. This phenomena is due to an *amplitude factor* appearing in the stationary-phase result, which alters the amplitude of the GO field. Essentially, the GO field now varies as

$$|E_z| \approx \frac{1}{\sqrt{x_n}} e^{(\alpha_z z) x_n}. \qquad (5)$$

This term is not predicted by the ray model, but is contained in the GO evaluation of the field. The important conclusion is that it is never possible to observe a leakage field that matches with the simple ray model, even when the asymptotic limit is approached so that the GO result is accurate.

C. Line in Free Space: Cross-Pol

In Figure 6, the cross-pol field is shown versus x_n with $z / \lambda_0 = 10^3$ and $p = 0.1, 0.5, 1.0,$ and 2.0.

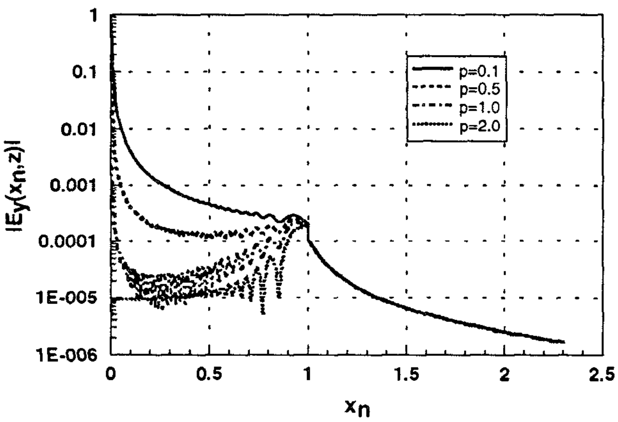

Fig. 6. The cross-pol field of the semi-infinite line source in free space versus x_n for $p = \alpha_z z / (2\pi) = 0.1, 0.5, 1.0,$ and 2.0, with $z / \lambda_0 = 1000$, $k / k_0 = 1.0$, and $\beta / k_0 = 0.967$. The electric field is evaluated at $y = -h$ and is normalized by multiplying with the distance $h = 0.1 \lambda_0$.

The field in this case begins with a behavior that has no exponential increase at all for the $p = 0.1$ case, then gradually begins to show some signs of an increasing field as p increases, but never becomes a purely exponential type of field. In the cross-pol case, the GO field varies as

$$|E_z| \approx \frac{1}{\sqrt{x^3_n}} e^{(\alpha_z z) x_n}. \qquad (6)$$

due to a different amplitude factor, which has an even greater influence on the leakage behavior than in the co-pol case.

V. CONCLUSIONS

It was found that for practical-length microwave transmission lines, the leakage fields exhibit a behavior unlike that predicted by the simple ray model of leakage, which predicts a smooth, exponentially increasing field within the lit region ($\theta < \theta_0$). In particular, the fields of the semi-infinite air-gap stripline exhibit a leakage behavior consistent with the ray model only in the asymptotic GO limit (i.e., $z / \lambda_0 \rightarrow \infty$ while $(\alpha_z z)$ remains finite). For smaller values of z / λ_0 the leakage fields look nothing like a simple exponential field. It was also found that the semi-infinite strip of current in free space *under no circumstances* has a leakage field that resembles a purely exponential field, even in the GO limit, due to an amplitude term that appears in the asymptotic expression for the field in the lit region.

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